Operating Leverage and Risk Premium *

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Abstract

We demonstrate that the operating leverage effect induced by fixed costs is affected by variable costs in firm production. This motivates us to propose two measures of firm-level operating leverage: a theoretically driven measure from a production-based model, and a measure from machine learning estimation. Both measures outperform the operating leverage measures in the existing literature in capturing the elasticity of operating profits with respect to gross profits. Furthermore, the operating leverage and risk premium relation depends on firms' gross profitability. For more profitable firms, fixed cost induces higher risk premium, consistent with the conventional wisdom on the operating leverage effect. For sufficiently low gross profitability firms, however, the operating hedge from variable costs creates a negative relation between operating leverage and risk premium. Our result poses a challenge for the explanation of the value premium relying on operating leverage.

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1 Introduction

Operating leverage is a fundamental concept in accounting, economics, and finance that measures the degree to which a project or firm committed to fixed production cost to generate profits. Conventional wisdom is that operating leverage tends to amplify the risk of a firm due to the stickiness of fixed cost and thus increases its expected return. Indeed, this channel has been widely used to explain the value premium (e.g., Carlson, Fisher, and Giammarino (2004), Zhang (2005)). However, empirical evidence for this mechanism is limited in scope and depth, partly because there is no consensus on how to accurately measure operating leverage in the literature.

In this paper, we propose two firm-level measures of operating leverage. The first measure, $\mathrm{OL}_{\mathrm{FL}}$, is motivated from a production-based asset pricing model and is defined as the ratio of the selling, general, and administrative expenses (SG&A) to gross profit. Unlike existing measures in the literature which we discuss further below, this measure is simple to construct and is entirely flow-based. The second measure, $\mathrm{OL}_{\mathrm{NN}}$, is statistical and estimated from a neural network with 140 firm characteristics. The use of the machine learning technique allows us to capture potential non-linear relation between operating leverage and firm characteristics as well as their interactions. We find the new measures are positively correlated with but significantly outperform the existing measures in the literature in capturing the elasticity of operating profits with respect to gross profits. Thus, our new measures more truthfully represent the operating leverage effect.

To motivate the flow-based operating leverage measure, we study a static value optimization problem of a firm with three types of production inputs: physical capital (such as properties, plants, and equipments or PPE), fixed inputs (e.g., SG&A), and variable inputs (e.g., COGS), with a nested constant elasticity of substitution (CES) production function. Following the literature on production functions, we first nest physical capital and fixed inputs and then nest this combined input with variable inputs, which allow the elasticity of substitution to differ among the production inputs.[1] Taking input prices as given, firms choose fixed and variable inputs to maximize their values, which in the static setting are equal to operating profits. We show that accounting variables such as gross margin and flow-based operating leverage (i.e., fixed costs divided by gross profits) naturally emerge from the first order conditions of firm's optimization problem.

It should be noted that considering the joint effect of fixed costs and variable costs on asset pricing is novel to the literature. Existing studies on asset pricing implications of

[1] This structure has been confirmed as a good approximation of the production behavior in several studies. See, for example, Carlstrom and Fuerst (2006), Bodenstein, Erceg, and Guerrieri (2011), and Kemfert (1998).

operating leverage only focus on fixed costs. However, variable costs, as documented in a recent paper by Kogan, Li, and Zhang (2023), are found to be more cyclical than revenues and create an operating hedge effect, contributing to the gross profitability premium (Novy-Marx (2013)). Indeed, by aggregating firm data from Compustat, we find that the elasticity of aggregate COGS with respect to the aggregate revenue is significantly larger than one (1.05). In contrast, the elasticity of SG&A is significantly lower than one (0.48) (see Table 1). Therefore, with firms optimally choosing the amount of fixed and variable inputs, this setup incorporates the interaction between operating leverage and operating hedge effects.

[Insert Table 1 Here]

Our production-based model shows that the operating leverage effect induced by fixed costs on a firm's risk exposure depends on the firm's gross margin (or gross profitability). When a firm's gross profitability is high, fixed costs raise the exposure of operating profits (gross profits minus fixed costs) to the aggregate profitability shock relative to gross profits, giving rise to an operating leverage effect. However, when a firm's gross profitability is sufficiently low, the fixed cost reinforces the operating hedge from variable inputs and further lowers the firm's risk premium.

We compare the flow-based measure $\mathrm{OL}_{\mathrm{FL}}$ with operating leverage measures currently used in the literature and examine the degree to which these measures capture the sensitivity of firms' operating profits to their gross profits. These existing operating leverage measures include the operating leverage defined in Novy-Marx (2011) ($\mathrm{OL}_{\mathrm{NM}}$, the sum of COGS and SG&A divided by AT), Chen, Chen, Li, and Li (2022) ($\mathrm{OL}_{\mathrm{CCLL}}$, the sum of DP and SG&A divided by market value of assets), Chen, Harford, and Kamara (2019) ($\mathrm{OL}_{\mathrm{CHK}}$, SG&A divided by AT), Ferri and Jones (1979) ($\mathrm{OL}_{\mathrm{FJ}}$, PPENT divided by AT). Our empirical evidence indicates that the new flow-based measure overwhelmingly dominates the alternative measures in capturing their elasticities of operating profits with respect to gross profits. Economically, a one-standard-deviation increase in $\mathrm{OL}_{\mathrm{FL}}$ raises the elasticity of operating profits with respect to gross profits by 1.14. As a comparison, a one-standard-deviation increase in the strongest operating leverage measure from existing studies, that is, $\mathrm{OLL}_{\mathrm{CCLL}}$ from Chen, Chen, Li, and Li (2022), is only associated with an increase in gross profit elasticity by 0.75. In a direct horse race between these two measures, the coefficient of $\mathrm{OL}_{\mathrm{FL}}$ decreases slightly to 1.08, whereas the coefficient of $\mathrm{OL}_{\mathrm{CCLL}}$ is reduced by half to only 0.38.

Despite being easy to construct, the flow-based operating leverage, $\mathrm{OL}_{\mathrm{FL}}$, has its limitation in that it is motivated from a static model which fails to take into account a firm's dynamic trade-offs. Furthermore, an operating leverage effect should depend not only on the levels of fixed cost and gross profit, but also their relative cyclicalities. To address these limitations and allow for substantially more flexibility, we utilize neural network to construct our second operating leverage measure, i.e., $\mathrm{OL}_{\mathrm{NN}}$. We find that $\mathrm{OL}_{\mathrm{NN}}$ captures most firm-level variations in operating profits sensitivity to gross profits. A one-standard-deviation increase in $\mathrm{OL}_{\mathrm{NN}}$ raises the elasticity of operating profits with respect to gross profits by 1.75. In a horse race analysis, $\mathrm{OL}_{\mathrm{NN}}$ subsumes all other operating leverage measures, including our flow-based measure $\mathrm{OL}_{\mathrm{FL}}$.

With the two new firm-level operating leverage measures, we empirically test the model's implications on the relationship between operating leverage and risk premium. In portfolios double-sorted on firms' gross profitability (GPA) and operating leverage, our results demonstrate that although the GPA premium (the difference in the average return between high and low GPA firms) remains positive across all operating leverage levels, it is more prominent among firms with high operating leverage. The GPA premium increases from 6.04% for firms in the low $\mathrm{OL}_{\mathrm{NN}}$ tercile to 9.45% for firms in the high $\mathrm{OL}_{\mathrm{NN}}$ terceile, and the result is even stronger when the flow-based measure $\mathrm{OL}_{\mathrm{FL}}$ is used. More importantly, we find the operating leverage is positively associated with average stock returns for high gross profit firms, but becomes negatively associated with average returns for firms with low gross profitability. This result confirms the prediction of the production-based model (i.e., Equation (7)) on the interaction of fixed and variable costs.

As we discussed earlier, one popular explanation for the value premium in the asset pricing literature is operating leverage (e.g., Carlson, Fisher, and Giammarino (2004), Zhang (2005)). This explanation relies on the intuition that firms with low productivity have low valuation ratio and high operating leverage and hence high risk premium, generating a negative cross-sectional correlation between valuation ratio and risk premium. With the newly proposed and more accurate operating leverage measures, we re-evaluate the contribution of the operating leverage effect on the value premium. Surprisingly, we find only a slightly positive correlation between book-to-market equity ratio and the two new measures of operating leverage. In addition, the value premium is strengthened, not weakened, by controlling for the new operating leverage measures. This latter result suggests that the operating leverage effect induced by fixed costs is unlikely to be the main source of the value premium.

Our paper is closely related to the literature on the effects of operating leverage and operating hedge on asset pricing. There are a large number of existing studies focusing on firms' operating leverage and its effects on stock returns. For instance, Carlson, Fisher, and Giammarino (2004) and Zhang (2005) show how operating leverage can generate a value spread in a neoclassical model of firm investment. Novy-Marx (2011) proposes an empirical measure of operating leverage and documents its positive predictive power for cross-sectional stock returns. A recent strand of related literature focuses on the effects of labor costs on stock returns, emphasizing wage rigidity as a source of operating leverage. For instance, Danthine and Donaldson (2002) show that wage rigidity can induce a strong labor leverage and improve the performance of asset pricing models with production to better match aggregate market volatility and equity premium. Favilukis and Lin (2015) examine the quantitative effect of wage rigidity and labor leverage on both the equity premium and the value premium. Donangelo, Gourio, Kehrig, and Palacios (2019) document that firms with high labor shares have higher expected returns than firms with low labor shares. In a new direction of exploration beyond the operating leverage, Kogan, Li, and Zhang (2023) uncover the importance of variable inputs in lowering a firm's risk premium, stemming from an operating hedge effect. They demonstrate that operating hedge is important in understanding the gross profitability premium in Novy-Marx (2013). Existing literature however only separately explored the operating leverage and the operating hedge effect. To the best of our knowledge, we are the first to evaluate a large number of operating leverage measures and examine the operating leverage effect on risk premium from both theoretical and empirical perspectives.

The paper proceeds as follows. In Section 2, we propose a model of firm production to demonstrate that fixed-cost induced operating leverage can be affected by the presence of variable costs. In Section 3, we construct the flow-based operating leverage measure motivated from the production-based model and the neural network based operating leverage measure, and compare them with various measures of operating leverage used in existing studies. In Section 4, we investigate the relationship between operating leverage and risk premium and evaluate the contribution of operating leverage in the value premium. We conclude in Section 5.

2 Firm Production and Operating Leverage

To illustrate how fixed and variable costs interact to determine a firm's risk premium, we consider a static production-based model.

2.1 The model

The economy is populated by a large number of profit-maximizing firms. Each firm produces its output (Y) using three inputs: physical capital (K), fixed inputs (A), and variable inputs (M). Physical capital includes properties, plants, and equipments. Examples of fixed costs include sales, general and administrative (SG&A) expenses such as rent and executive compensation. Variable inputs include all inputs directly used in a firm's production process such as materials, intermediate goods and services, typically reflected in costs of goods sold (COGS).

We assume a constant elasticity of substitution (CES) production function. Following the literature on production functions with multiple inputs, we adopt a nested specification by first combining physical capital (K) and fixed inputs (A) to obtain integrated inputs (V) with a constant elasticity of substitution ρ between K and A. We then combine integrated inputs (V) and variable inputs (M) with a constant elasticity of substitution of θ. Specifically, firm's output Y is given by

$$Y = \left((ZM)^{\frac{\theta-1}{\theta}} + \left\{ \left[(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \right\}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

where U and Z represent idiosyncratic productivity on fixed and variable inputs, respectively, and X is the capital-augmenting aggregate productivity. Let V denote firm's integrated inputs by combining physical capital K and fixed inputs A, that is,

$$V = \left[(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}. \tag{2}$$

Firm's output Y can then be expressed as

$$Y = \left[(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

Firms in our economy own physical capital, so their objective is to maximize operating profits OP by choosing variable inputs M and fixed inputs A. That is,

$$OP = \max_{M,A} GP - P_A A = \max_{M,A} Y - P_M M - P_A A, \tag{4}$$

where GP is gross profit, and P_M and P_A are the prices of variable and fixed inputs, respectively.

From the derivation in the appendix, we have the following two results. First, the operating hedge effect from Kogan, Li, and Zhang (2023) can be observed from the difference in the exposure of gross profits and outputs to the aggregate profitability shock. Denoting $p_1^M \equiv \frac{\partial \log P_M}{\partial \log X}$ as the cyclicality of variable input price with respect to aggregate profitability X, and $GM \equiv 1 - \frac{P_M M}{Y}$ as the gross margin, we have the following proposition.

PROPOSITION 1. *With the production technology described above, the difference in the exposure of gross profits and outputs to the aggregate profitability shock, i.e., the operating* *hedge effect, is given by*

$$\frac{\partial \log GP}{\partial \log X} - \frac{\partial \log Y}{\partial \log X} = p_1^M(\theta - 1)\frac{1 - GM}{GM}. \tag{5}$$

Proof: See the Appendix.

Proposition 1 suggests that when $\theta < 1$ and $p_1^M > 0$, a condition that is empirically confirmed in Kogan, Li, and Zhang (2023), the variable cost always reduces the firm's systematic risk. In other words, the operating hedge effect exists regardless if we model fixed inputs in the production function. Furthermore, the strength of operating hedge decreases with gross margin, so that more profitable firms are associated with lower operating hedge effect. This finding is consistent with the explanation in Kogan, Li, and Zhang (2023) for the gross profitability premium.

The second result is new and represents the effect of fixed costs on risk premium, which can be observed from the difference between the exposures of operating profits and gross profits to the aggregate profitability shock. Denoting $p_1^A \equiv \frac{\partial \log P_A}{\partial \log X}$ as the cyclicality of fixed input price with respect to aggregate profitability X, and $OL \equiv \frac{P_A A}{GP}$ as the ratio of fixed cost to gross profit, we have the following proposition regarding the operating leverage effect.

PROPOSITION 2. *With the production technology described above, the difference in the exposure of operating profits and gross profits to the aggregate profitability shock, i.e., the operating leverage effect, is given by*

$$\frac{\partial \log OP}{\partial \log X} - \frac{\partial \log GP}{\partial \log X} = (1 - \rho)\frac{OL}{1 - OL}\left[\left(p_1^M - p_1^A\right) - \frac{p_1^M}{GM}\right]. \tag{6}$$

Proof: See the Appendix.

Proposition 2 illustrates the relation between the operating leverage effect and firms' gross profitability. When $\rho < 1$ and $p_1^A < 0 < p_1^M$, a condition that is the consistent with the literature and also confirmed in the data, the effect of fixed costs on the risk premium depends on the firm's gross margin GM. When gross margin is high, the term in the square bracket in Equation (6) is positive, so the conventional operating leverage effect exists. For instance, when the model abstracts from variable inputs (i.e., $GM = 1$), an assumption that is usually made in models that study operating leverage, the right-hand-side of Equation (6) is positive and increases in OL, and fixed cost raises a firm's risk premium. However, when the gross margin is sufficiently low, $\left[\left(p_1^M - p_1^A\right) - \frac{p_1^M}{GM}\right]$ can turn negative. In this case, fixed costs lower, not raise, the risk premium. The intuition is that for firms with sufficiently low gross margin, the operating hedge from variable costs becomes much stronger so that the gross profits become less risky (i.e., have lower aggregate profitability beta) than fixed costs, and any fixed cost will reduce the firm's systematic risk even further.

Note OL measures how fixed cost amplifies the risk of gross profits and is therefore our theoretically motivated measure of operating leverage used in our subsequent empirical analysis. Because both the denominator and numerator in the OL definition are variables from income statements, we refer to it as a flow-based operating leverage and denote it as OL_{FL} in the next section.

The above intuition extends to the firm's overall risk exposure to the aggregate profitability shock, as summarized in the following proposition.

PROPOSITION 3. *With the production technology described above, the firm's risk exposure to the aggregate profitability shock is given by*

$$\beta \equiv \frac{\partial \log OP}{\partial \log X} = 1 + p_1^A + \frac{1}{1 - OL}\left[\left(p_1^M - p_1^A\right) - \frac{p_1^M}{GM}\right]. \quad (7)$$

Proof: See the Appendix.

Proposition 3 has three implications. First, when the variable input price is procyclical, i.e., $p_1^M > 0$, holding the firm's operating leverage (OL) constant, a firm's beta to the aggregate profitability shock (β) increases in firm's gross margin (GM) or gross profitability. Therefore, high profitability firms have higher exposure to the aggregate profitability shock at a given level of operating leverage. This offers an explanation for the gross profitability premium. Second, the positive relation between gross profitability and aggregate profitability beta is stronger among firms with high OL. This suggests that operating leverage may amplify the gross profitability premium. Third, the relation between aggregate profitability beta and operating leverage can be positive or negative depending upon gross profitability. For firms with high profitability, their exposure to the aggregate profitability shock increases in the firm's operating leverage. When firm's gross profitability is low, the relationship between exposure to the aggregate profitability shock and operating leverage becomes negative. This implies an intricate relation between firms' operating leverage and risk premium.

2.2 Value and Policy Functions

Table 2 lists the parameter values we use for the model calibration, and the Figure 1 plots the firm's optimal fixed input (A) and variable input (M), gross profitability (GP/A), operating leverage (OL), gross margin (GM), and operating profitability (OP/A), against the firm-level productivity of fixed inputs (u) and variable inputs (z).

[Insert Table 2 Here]

[Insert Figure 1 Here]

The top left and middle panels of Figure 1 show that the firm's optimal fixed inputs and variable inputs both increase with the productivity of variable inputs (z). However, their relation to the fixed input productivity (u) is more complex. While there is always a positive relation between the variable inputs (M) and the fixed input productivity (u), the relation between the optimal fixed inputs (A) and the fixed input productivity (u) depends upon the level of the variable input productivity (z). When the variable input productivity (z) is low, the optimal fixed inputs (A) increase in fixed input productivity (u). At high level of the variable input productivity, the optimal fixed inputs (A) decrease in the fixed input productivity (u). More generally, the relation between the optimal fixed inputs (A) and the fixed input productivity (u) can be non-monotonic.

The top right and bottom left panels of Figure 1 plots how a firm's gross profitability (GP/A) and operating leverage (OL), respectively, vary with the variable input productivity (z) and the fixed input productivity (u). While firm gross profitability is mostly driven by the variable input productivity (z), a firm's operating leverage is affected by both its variable input productivity (z) and fixed input productivity (u). Firms with both low variable input and fixed input productivities have high operating leverage. The bottom middle panel of Figure 1 shows that a firm's gross margin only depends on its variable input productivity (z). Therefore, gross profitability and gross margin are strongly correlated in the model. The bottom right panel plots the operating profitability (the firm value in our economy) against these two idiosyncratic input productivities. Despite a similar pattern to that of the gross profitability (top right panel), we find operating profitability (OP/A) demonstrates a stronger relation to the fixed input productivity (u) than gross profitability (GP/A). This is especially true at high variable input productivity, i.e., high gross profitability.

An important question for asset pricing is how the risk premium varies across firms. Given the focus of our study, we are particularly interested in the relation of a firm's risk premium to its gross profitability and operating leverage. The top panel of Figure 2 shows the relation of the firm's aggregate profitability shock exposure (β) to the fixed input productivity (u) and the variable input productivity (z). We find that the firm's exposure to the aggregate profitability shock monotonically increases in its variable input productivity (z). However, the firm's aggregate profitability shock exposure increases in the fixed input productivity (u) only when the variable input productivity (z) is low. The relation reverses when the firm's variable input productivity (z) is high with the firm's aggregate risk exposure decreases in the fixed input productivity.

[Insert Figure 2 Here]

More important, when we plot the firm's aggregate profitability shock exposure against the firm's gross profitability (GP/A) and operating leverage (OL) in the bottom panel of Figure 2, the following patterns emerge. First, the firm's risk exposure to the aggregate profitability shock increases in firm's gross profitability at all levels of the firm's operating leverage. Therefore, our model predicts a positive gross profitability premium. Furthermore, the profitability premium is stronger for firms with high operating leverage. In contrast, the relation between risk exposure and operating leverage depends on the firm's gross profitability. Specifically, the firm's risk exposure slightly increases in its operating leverage at high level of firm profitability, but decreases in its operating exposure at low level of firm profitability. This is consistent with the relation between firm's risk exposure to aggregate profitability shock and its operating leverage OL as shown in Equation (7). We test these predictions using characteristic-sorted portfolios in the next section.

3 Measuring Operating Leverage

In this section, we discuss the construction of our two new measures of operating leverage, and compare them with the measures used in the existing literature.

3.1 Construction of operating leverage measures

The first proposed measure is the flow-based operating measure derived from our production-based model in the previous section, OL_{FL}. Specifically, we define OL_{FL} as the ratio of selling, general, and administrative expenses (Compustat item XSGA) to gross profits (Compustat item GP). There are several operating leverage measures proposed in existing literature. These include the measure used in Novy-Marx (2011) (defined as the ratio of the sum of COGS and SG&A to total asset (AT) and denoted as OL_{NM}), the measure from Chen, Chen, Li, and Li (2021) (defined as the ratio of the sum of depreciation (DP) and SG&A to market value of assets and denoted as OL_{CCLL}), the measure from Chen, Hartford, and Kamara (2019) (defined as the ratio of SG&A to total asset (AT) and denoted as OL_{CHK}), and the measure from Ferri and Jones (1979) (defined as the ratio of net property, plant and equipment (PPENT) to total asset (AT) and denoted as OL_{FJ}).

There are two major differences between $\mathrm{OL_{FL}}$ and the existing measures from the literature. First, we differentiate cost of goods sold (Compustat item COGS) and SG&A expenses. As discussed in the introduction, these two types of costs have different cyclicality with respect to outputs. Thus, they should be treated differentially in studying their implications for asset prices. The concept of operating leverage is more appropriate for the operating costs that are relatively "sticky" such as SG&A, which is the numerator of our measure. Second, $\mathrm{OL_{FL}}$ is flow-based, and its denominator is gross profit (the item right above SG&A in income statements). Again, this choice of denominator is consistent with the theoretical model discussed above and with the convention that operating leverage is associated with fixed costs driving up cash flow risks. In contrast, except for $\mathrm{OL_{CCLL}}$, all other measures use total asset (Compustat item AT) as the denominator.

Our second measure, $\mathrm{OL_{NN}}$, is purely statistical and is aimed at extracting information from possible determinants of firm-level operating leverage that are missed by $\mathrm{OL_{FL}}$. For example, the flow-based measure abstracts from a firm's intertemporal trade-offs in a dynamic setting; it ignores the heterogeneity in cylicalities of fixed input prices across firms; it also fails to capture the tension between financial leverage and operating leverage (see, e.g., Favilukis, Lin, and Zhao (2020)). To allow for sufficient flexibility to measure a firm's operating leverage, we use the neural network machine learning technique.

Specifically, we estimate the following model:

$$\%\Delta OP_{i,t} = \widehat{OL}_{i,t}(X_{i,t}^{(\cdot)}) \times \%\Delta GP_{i,t}, \tag{8}$$

where $\widehat{OL}_{i,t}$ is the fitted operating leverage from firm characteristics, $X_{i,t}^{(\cdot)}$, that best describes the sensitivity of operating profit with a change in gross profit. Our choice of $X_{i,t}^{(\cdot)}$ includes 140 characteristics from a total of 212 stock return predictors summarized in Chen and Zimmermann (2021) that are compatible with our sample. Of the 140 characteristics, 117 are continuous variables, so we winsorize them cross-sectionally at 1% and 99%. For each characteristic, we impute missing values with its cross-sectional median. We further standardize these characteristics to have zero means and unit standard deviations before implementing the neural network estimation. The custom loss function is expressed as follows

$$\mathcal{L} = \frac{1}{N \cdot T} \sum_{i,t=1}^{N,T} \left[\left(\%\Delta OP_{i,t} - \hat{OL}_{i,t} \times \%\Delta GP_{i,t} \right) - \frac{1}{N \cdot T} \sum_{i,t=1}^{N,T} \left(\%\Delta OP_{i,t} - \hat{OL}_{i,t} \times \%\Delta GP_{i,t} \right) \right]^2 \tag{9}$$

We model $\widehat{OL}_{i,t}(X_{i,t}^{(\cdot)})$ as the exponential of a neural network of $X_{i,t}^{(\cdot)}$.[2] For simplicity, we build our neural network with an input layer (64 neurons) and one hidden layer (32 neurons) with rectified linear unit (ReLU) activation function, and one output layer with a linear activation function, which generates continuous predicted values. Within each layer, we use He initialization to normalize weights and apply l_1 penalty to regularize over-fitting. We use batch normalization to stabilize the training process and compile the model using Adam optimizer. We train the model in the training sample and compare the performance in both training and testing sample. There are three hyperparameters in the estimation: (1) strength of l_1 regularization; (2) learning rate, which regulates the convergence speed in Adam optimizer; and (3) proportion for train-test split. We use cross-validations to select these hyperparameters.[3]

3.2 Comparison among operating leverage measures

Table 3, Panel A summarizes the correlation matrix of different operating leverage measures. Most of these operating leverage measures are positively correlated, with one notable exception of OL_{FJ} from Ferri and Jones (1979), which has negative correlations with all other measures. For our flow-based measure OL_{FL}, the correlation is 0.32 with OL_{NM} from Novy-Marx (2011), 0.61 with OL_{CHK} from Chen, Hartford, and Kamara (2019), and 0.53 with OL_{CCLL} from Chen, Chen, Li, and Li (2021). For the neural network based measure OL_{NN}, the correlation is 0.09 with OL_{NM}, 0.29 with OL_{CHK}, and 0.32 with OL_{CCLL} from Chen, Chen, Li, and Li (2021). The correlation between OL_{FL} and OL_{NN} is 0.68, indicating that the flow-based measure correlates the most with OL_{NN} and explains a large fraction of the variation in OL_{NN}.

[Insert Table 3 Here]

Panel B of Table 3 reports the correlation between the two new operating leverage measures and firm characteristics including logarithm of firm size (logME), logarithm of book-to-market equity ratio (logBM), gross profitability (GPA), and idiosyncratic volatility (IVOL). In general, firms with high operating leverage are those with small market cap and high idiosyncratic volatility. The correlation between operating leverage and GPA depends on the specific measures: while the flow-based measure is strongly and positively correlated with GPA (0.34), the correlation between OL_{NN} and GPA is much smaller. Lastly, the correlation

[2] We choose this specification to avoid negative fitted operating leverage. The result is very similar when we directly model operating leverage as a neural network of $X_{i,t}^{(\cdot)}$.

[3] The selected values for the hyperparameters are: l_1 regularization parameter = 0.0001, learning rate = 0.01, and fraction of testing sample = 0.4.

between operating leverage and logBM is only slightly positive for both measures of operating leverage. We will return to understanding the contribution of operating leverage to the value premium in Section 4.

We next estimate firm-level elasticities of operating profits with respect to gross profits and study how these elasticities vary with these operating leverage measures in Table 4. Specifically, we run panel regressions of percentage change in operating profits onto the firm-level gross profit growth and their interaction with all measures mentioned above. The measure with the largest coefficient on the interaction term and largest R^2 represents the highest gross profit elasticity of firms' operating profit, thus best captures the operating leverage effect. Specification (1) provides a benchmark that assumes constant elasiticty across all firms. The result shows that unconditionally, a one-percent increase in gross profits is associated with around 5% increase in operating profits, and even the homogeneity assumption can explain 70.5% of the variation in percentage change in operating profits. Specifications (2)-(7) of Table 4 include one operating leverage measure at a time and the results show a large heterogeneity in elasticity across firms, with the elasticity larger among firms with high $\mathrm{OL}_{\mathrm{FL}}$, $\mathrm{OL}_{\mathrm{NN}}$, $\mathrm{OL}_{\mathrm{NM}}$, $\mathrm{OL}_{\mathrm{CCLL}}$, and $\mathrm{OL}_{\mathrm{CHK}}$, but low $\mathrm{OL}_{\mathrm{FJ}}$. Therefore, all six operating leverage measures except $\mathrm{OL}_{\mathrm{FJ}}$ capture some degree of the operating leverage effect. Economically, a one-standard-deviation increase in $\mathrm{OL}_{\mathrm{FL}}$, $\mathrm{OL}_{\mathrm{NN}}$, $\mathrm{OL}_{\mathrm{NM}}$, $\mathrm{OL}_{\mathrm{CHK}}$, and $\mathrm{OL}_{\mathrm{CCLL}}$ is associated with an increase in the gross profit elasticity of operating profits by 1.14, 1.75, 0.33, 0.67, and 0.75, respectively, and a one-standard-deviation increase in $\mathrm{OL}_{\mathrm{FJ}}$ is associated with a decrease in the elasticity by 0.35. The inclusion of our flow-based measure can increase the R^2 to 78.8% and the inclusion of the neural network based measure can raise the R^2 to 90.3%. As a comparison, the maximum R^2 is at 72.3% for all existing measures.

[Insert Table 4 Here]

In Specifications (8)-(12), we conduct horse races between the flow-based measure $\mathrm{OL}_{\mathrm{FL}}$ with the existing operating leverage measures. The result shows that the inclusion of $\mathrm{OL}_{\mathrm{FL}}$ significantly reduces the coefficients of other operating leverage measures, whereas the coefficient on $\mathrm{OL}_{\mathrm{FL}}$ is only slightly affected. The result is even stronger for the neural network based measure $\mathrm{OL}_{\mathrm{NN}}$ in Specifications (13)-(17), where we find the coefficient on the existing measures becomes either statistically insignificant or turns negative with the inclusion of $\mathrm{OL}_{\mathrm{NN}}$. Specification (18) compares the two new operating leverage measures $\mathrm{OL}_{\mathrm{FL}}$ and $\mathrm{OL}_{\mathrm{NN}}$. While $\mathrm{OL}_{\mathrm{FL}}$ dominates the existing measures, it is subsumed by $\mathrm{OL}_{\mathrm{NN}}$ and the coefficient of $\mathrm{OL}_{\mathrm{FL}}$ becomes very close to zero. Specification (19) includes all measures and the result shows that $\mathrm{OL}_{\mathrm{NN}}$ is clearly superior to all other measures in capturing the cross-sectional heterogeneity in the elasticity of operating profits with respect to gross profits.

4 Operating leverage and risk premiums

We now turn our attention to the relation between firms' operating leverage and risk premium. Motivated by firms' differential risk exposures to aggregate profitability shocks with respect to their operating leverage and gross profitability, we form portfolios sorted by their operating leverage and gross profitability. We then investigate how firms' operating leverage affects their risk premium by examining the pattern of average stock returns of the constructed portfolios. Furthermore, we also examine how the operating leverage effect contributes to the value premium. The stock return data are from the Center for Research in Security Prices (CRSP) database. We only include stocks with share code (CRSP item SHRCD) of 10 or 11, and exchange code (CRSP item EXCHCD) of 1, 2, or 3. We exclude firms in the financial industry (SIC between 6000 and 6999) and utility industry (SIC between 4950 and 4999). Our benchmark sample is from July 1964 to June 2020.

4.1 Double sorted portfolios

We begin our tests with portfolios double sorted on gross profitability and operating leverage. The theoretical analysis in Section 2 shows that the gross profitability premium is stronger for firms with high operating leverage. Furthermore, the effect of operating leverage on a firm's risk premium depends on its gross profitability. For high gross profit firms, the conventional wisdom holds and high operating leverage firms are riskier. However, when gross profitability is sufficiently low, the operating hedge induced by variable costs generates a negative relation between operating leverage and risk premium.

Table 5 Panel A presents the results on the average stock returns for 3-by-5 portfolios sequentially sorted by $\mathrm{OL}_{\mathrm{FL}}$ and GPA (Panel A 1)) and 3-by-5 portfolios sequentially sorted by GPA and $\mathrm{OL}_{\mathrm{FL}}$ (Panel A 2)). Controlling for $\mathrm{OL}_{\mathrm{FL}}$, the GPA premium is large in magnitude and statistically significant. Further, the GPA premium is much larger among firms with high operating leverage. The GPA premium is 2.62% for firms in the lowest $\mathrm{OL}_{\mathrm{FL}}$ tercile and increases to 8.78% for firms in the highest $\mathrm{OL}_{\mathrm{FL}}$ tercile. Consistent with the model prediction, we also find the $\mathrm{OL}_{\mathrm{FL}}$ premium is negative for firms with low gross profitability (-3.13% per year for firms in the lowest GPA tercile) and becomes positive for firms with high gross profitability (4% per year for firms in the highest GPA tercile). The results using the neural network based operating leverage measure is qualitatively the same, as shown in Panel B of Table 5. The GPA premium is larger ranging from 6.04% for the lowest $\mathrm{OL}_{\mathrm{NN}}$ tercile firms to 9.45% for the highest $\mathrm{OL}_{\mathrm{NN}}$ tercile firms and all statistically significant. The $\mathrm{OL}_{\mathrm{NN}}$ premium shows a tighter range from -1.67% for the lowest GPA tercile firms to 3.64% for the highest GPA tercile firms.

[Insert Table 5 Here]

Taken together, the interdependence of gross profitability, operating leverage, and average stock return lends support to the prediction of our production-based model and provides strong empirical evidence on fixed-cost induced operating leverage effect being affected by the variable-cost driven operating hedge effect. The finding also manifest the importance and imperativeness of simultaneously taking into account fixed and variable costs in production-based asset pricing models.

4.2 Univariate portfolio sort

We now examine decile portfolios one-way sorted on gross profitability and the two new measures of operating leverage to study the unconditional premiums associated with these firm characteristics. The results are reported in Table 6.

On the portfolios sorted by firm gross profitability, the GPA premium remains positive regardless of the level of operating leverage, and we expect a strong gross profitability premium. We confirm this in Table 6 Panel A which reports the average excess stock returns, alphas and factor betas from the unconditional CAPM and Fama and French three-factor model for decile portfolios sorted on gross profitability. We observe a large and significant spread in GPA, CAPM alpha, and Fama-French-three-factor alpha (6.28%, 7.95%, and 9.90%, respectively) between high-minu-low GPA portfolios, which are consistently statistically significant (t-statistic of 2.77, 3.50, and 4.75, respectively). High GPA firms have a negative loading on the HML, lending support to the argument in Novy-Marx (2013) that profitability premium is the other side of value. Indeed, accounting for the HML factor in return regressions increases the spread in alpha as the Fama-French-three-factor alpha being almost two percentage points larger than the CAPM alpha.

[Insert Table 6 Here]

The dependence of the operating leverage premium on gross profitability from the previous subsection suggests that the unconditional operating leverage premium may not be monotonic. In particular, firms with very high operating leverage are those with low profitability, so the operating hedge from variable costs can lower their systematic risks, giving rise to a hump-shaped relation between operating leverage and risk premium. We find this is indeed the case in the data. In Panel B of Table 6 when we sort firms into decile portfolios based on their $\mathrm{OL}_{\mathrm{FL}}$, the average excess return initially increases from 5.51% from decile 1 to 9.95% in decile 8, and then sharply decreases to 0.95% in decile 10. The long-short return spread between high and low $\mathrm{OL}_{\mathrm{FL}}$ is -4.56%. The exposures to the market and Fama and French three factors do not explain the hump-shaped pattern in the average returns, as the CAPM and Fama-French three-factor alphas also display hump shape. The long-short portfolio has even larger abnormal return in magnitude, with −7.9% for CAPM alpha and −7.63% for Fama and French three-factor alpha. Panel C of Table 6 reports the results for portfolios sorted on $\mathrm{OL}_{\mathrm{NN}}$. The overall patterns are the same as those from Panel B, although the size of the return spread is relatively smaller than that from $\mathrm{OL}_{\mathrm{FL}}$ sorts.

4.3 Operating leverage and value premium

Lastly, we explicitly assess the relation between operating leverage and the value premium. Panel A of Table 7 presents the results for one-way sorted portfolios based on book-to-market equity ratio (BM). For the sample period between July 1964 and June 2020, the conditional value premium is about 3.73%, which is only marginally significant with a t-statistic of 1.69. Value stocks tend to have higher operating leverage than growth stocks, a finding that is consistent with Table 3. Value firms also have low gross profitability and small firm size.

[Insert Table 7 Here]

Panels B and C of Table 7 examine the value premium conditional on our new operating leverage measures. For both $\mathrm{OL}_{\mathrm{FL}}$ and $\mathrm{OL}_{\mathrm{NN}}$, we find the value premium is stronger for firms with high operating leverage. More important, the average value premium across operating leverage terciles, or the conditional value premium, is substantially stronger than the unconditional value premium from Panel A. The value premium conditional on $\mathrm{OL}_{\mathrm{FL}}$ is 4.94% per year with a t-statistic of 3.73, and the value premium conditional on $\mathrm{OL}_{\mathrm{NN}}$ is 3.69% per year with a t-statistic of 2.84. The stronger conditional value premium suggests that the operating leverage effect is unlikely the main driving force for the value premium. If anything, firms' operating leverage tends to weaken the value premium. Our findings therefore pose a challenge to the explanation of value premium that relies on operating leverage as frequently argued in exiting literature.

5 Conclusion

We revisit a key concept widely used in finance, accounting, and economics literature—operating leverage. Because a firm's operating leverage measures the degree to which a firm can increase its operating profit with its gross profit, the conventional wisdom often directly associates a firm's operating leverage to its risk premium. Using a production-based model with three types of inputs: physical capital, fixed inputs, and variable inputs, we demonstrate that the fixed-cost induced operating leverage effect is intricately impacted by variable production cost. The problem is exacerbated by drastically different operating leverage measures used in existing studies which may even be negatively correlated with each other. It is thus crucial to identify the appropriate measures of operating leverage that best captures the sensitivity of a firm's operating profit to its gross profit.

We propose two new firm-level measures of operating leverage. The first measure is motivated from the product-based model and is defined as the selling, general, and administrative expenses (SG&A) divided by gross profit. This measure is simple to construct and is entirely flow-based. The second measure is estimated from a neural network with more than 100 firm characteristics. We find the new measures are positively correlated with but significantly outperform the existing measures in the literature in capturing the elasticity of operating profits with respect to gross profits.

More important, we find that the operating leverage effect on risk premium depends critically on firms' gross profitability. The operating leverage premium is positive for firms with high gross profitability, a pattern that is consistent with the conventional wisdom. However, for firms with sufficiently low gross profitability, the operating leverage premium becomes negative, due to the operating hedge from variable costs, as documented in Kogan, Li, and Zhang (2023). Our analysis also suggests that the well-known value premium is unlikely due to the difference in the operating leverage between value and growth firms.

References

Bodenstein, Martin, Christopher J Erceg, and Luca Guerrieri, 2011, Oil shocks and external adjustment, Journal of International Economics 83, 168–184.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, Journal of Finance 59, 2577–2603.

Carlstrom, Charles T, and Timothy Stephen Fuerst, 2006, Oil prices, monetary policy, and counterfactual experiments, Journal of Money, Credit, and Banking 38, 1945–1958.

Chen, Andrew Y, and Tom Zimmermann, 2021, Open source cross-sectional asset pricing, Critical Finance Review, Forthcoming.

Chen, Huafeng, Jason V Chen, Feng Li, and Pengfei Li, 2022, Measuring operating leverage, The Review of Asset Pricing Studies 12, 112–154.

Chen, Zhiyao, Jarrad Harford, and Avraham Kamara, 2019, Operating leverage, profitability, and capital structure, Journal of financial and quantitative analysis 54, 369–392.

Danthine, Jean-Pierre, and John B. Donaldson, 2002, Labour relations and asset returns, The Review of Economic Studies 69, 41–64.

Donangelo, Andres, Francois Gourio, Matthias Kehrig, and Miguel Palacios, 2019, The cross-section of labor leverage and equity returns, Journal of Financial Economics 132, 497–518.

Favilukis, Jack, and Xiaoji Lin, 2015, Wage rigidity: A quantitative solution to several asset pricing puzzles, The Review of Financial Studies 29, 148–192.

———, and Xiaofei Zhao, 2020, The elephant in the room: The impact of labor obligations on credit markets, American Economic Review 110, 1673–1712.

Ferri, Michael G, and Wesley H Jones, 1979, Determinants of financial structure: A new methodological approach, The Journal of finance 34, 631–644.

Kemfert, Claudia, 1998, Estimated substitution elasticities of a nested ces production function approach for germany, Energy Economics 20, 249–264.

Kogan, Leonid, Jun Li, and Harold H Zhang, 2023, Operating hedge and gross profitability premium, The Journal of Finance 78, 3387–3422.

Novy-Marx, Robert, 2011, Operating leverage, Review of Finance 15, 103–134.

———, 2013, The other side of value: The gross profitability premium, Journal of Financial Economics 108, 1–28.

Zhang, Lu, 2005, The value premium, Journal of Finance 60, 67–103.

Appendix

First order conditions

Specifically, firm's production function is given by

$$Y = \left((ZM)^{\frac{\theta-1}{\theta}} + \left\{ \left[(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \right\}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{A.1}$$

where U and Z represent idiosyncratic productivity shocks to the fixed inputs and variable inputs, respectively. Let V to denote the integrate capital by combining physical capital K and fixed inputs A, that is,

$$V = \left[(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{A.2}$$

Firm's output Y can then be expressed as

$$Y = \left[(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{A.3}$$

Firm maximizes its operating profit OP by choosing fixed inputs A and variable inputs M. That is,

$$OP = \max_{\{M,A\}} \{Y - P_M M - P_A A\} \tag{A.4}$$

where P_M and P_A are the prices of variable and fixed inputs, respectively.

The first order conditions are given by

$$\frac{\partial OP}{\partial M} = \left[(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}-1} Z^{\frac{\theta-1}{\theta}} M^{\frac{\theta-1}{\theta}-1} - P_M = 0$$
$$\Rightarrow P_M = \left[(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta-1}} Z^{\frac{\theta-1}{\theta}} M^{-\frac{1}{\theta}} \tag{A.5}$$

$$\frac{\partial OP}{\partial A} = \left[(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}-1} V^{\frac{\theta-1}{\theta}-1} \left[(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}-1} U^{\frac{\rho-1}{\rho}} A^{\frac{\rho-1}{\rho}-1} - P_A = 0$$
$$\Rightarrow P_A = \left[(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta-1}} \left[(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right]^{\frac{\theta-\rho}{(\rho-1)\theta}} U^{\frac{\rho-1}{\rho}} A^{-\frac{1}{\rho}} \tag{A.6}$$

Capital productivity $\frac{Y}{K}$

Multiplying both sides of equation (A.5) by $\frac{M}{Y}$ yields

$$\frac{P_M M}{Y} = \frac{(ZM)^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}}} = \frac{\left(\frac{ZM}{V}\right)^{\frac{\theta-1}{\theta}}}{\left(\frac{ZM}{V}\right)^{\frac{\theta-1}{\theta}} + 1} \tag{A.7}$$

Multiplying both sides of equation (A.6) by $\frac{A}{Y}$ yields

$$\begin{aligned}\frac{P_A A}{Y} &= \frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}}} \cdot \frac{(UA)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}} + (K)^{\frac{\rho-1}{\rho}}} \\ &= \frac{1}{\left(\frac{ZM}{V}\right)^{\frac{\theta-1}{\theta}} + 1} \cdot \frac{\left(\frac{UA}{XK}\right)^{\frac{\rho-1}{\rho}}}{\left(\frac{UA}{K}\right)^{\frac{\rho-1}{\rho}} + X^{\frac{\rho-1}{\rho}}}\end{aligned} \tag{A.8}$$

From equation (A.7) we have

$$\left(\frac{ZM}{V}\right)^{\frac{\theta-1}{\theta}} = \frac{P_M M}{Y - P_M M} \tag{A.9}$$

Plugging equation (A.9) into equation (A.8) gives

$$\left(\frac{UA}{XK}\right)^{\frac{\rho-1}{\rho}} = \frac{P_A A}{Y - P_M M - P_A A} \tag{A.10}$$

Dividing both sides of equation (A.1) by K gives the capital productivity $\frac{Y}{K}$

$$\begin{aligned}\frac{Y}{K} &= \left\{\left(\frac{ZM}{K}\right)^{\frac{\theta-1}{\theta}} + \left[\left(\frac{UA}{K}\right)^{\frac{\rho-1}{\rho}} + X^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}} \\ &= \left\{\left(\frac{ZM}{V}\right)^{\frac{\theta-1}{\theta}}\left(\frac{V}{K}\right)^{\frac{\theta-1}{\theta}} + \left[\left(\frac{UA}{K}\right)^{\frac{\rho-1}{\rho}} + X^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}\end{aligned} \tag{A.11}$$

Since equation (A.2) can also be expressed in per unit of capital term, that is,

$$\frac{V}{K} = \left[\left(\frac{UA}{K}\right)^{\frac{\rho-1}{\rho}} + X^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{A.12}$$

Plugging equation (A.12) into equation (A.11) gives

$$\frac{Y}{K}=\left\{\left(\frac{ZM}{V}\right)^{\frac{\theta-1}{\theta}}\left[\left(\frac{UA}{K}\right)^{\frac{\rho-1}{\rho}}+X^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}}+\left[\left(\frac{UA}{K}\right)^{\frac{\rho-1}{\rho}}+X^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}$$

$$=\left[\left(\frac{ZM}{V}\right)^{\frac{\theta-1}{\theta}}+1\right]^{\frac{\theta}{\theta-1}}\left[\left(\frac{UA}{XK}\right)^{\frac{\rho-1}{\rho}}+1\right]^{\frac{\rho}{\rho-1}} \tag{A.13}$$

Plugging equations (A.9) and (A.10) into equation (A.13) gives

$$\frac{Y}{K}=\left(\frac{Y-P_MM}{Y-P_MM-P_AA}\right)^{\frac{\rho}{\rho-1}}\left(\frac{Y}{Y-P_MM}\right)^{\frac{\theta}{\theta-1}}X \tag{A.14}$$

Exposure of firm inputs to aggregate profitability shock

The production function is augmented by three inputs K, A, and M. K is fixed in the static model, so we have

$$\frac{\partial\log K}{\partial\log X}=0 \tag{A.15}$$

$\frac{\partial\log A}{\partial\log X}$ and $\frac{\partial\log M}{\partial\log X}$ can be solved from taking partial derivative of the logarithm of both sides of equations (A.5) and (A.6), that is,

$$\frac{\partial\log P_M}{\partial\log X}=\frac{1}{\theta-1}\frac{\partial\log\left((ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}\right)}{\partial\log X}-\frac{1}{\theta}\frac{\partial\log M}{\partial\log X} \tag{A.16}$$

$$\frac{\partial\log P_A}{\partial\log X}=\frac{1}{\theta-1}\frac{\partial\log\left((ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}\right)}{\partial\log X}+\frac{\theta-\rho}{(\rho-1)\theta}\frac{\partial\log\left((UA)^{\frac{\rho-1}{\rho}}+(XK)^{\frac{\rho-1}{\rho}}\right)}{\partial\log X}-\frac{1}{\rho}\frac{\partial\log A}{\partial\log X} \tag{A.17}$$

We have that

$$\frac{\partial\log\left((UA)^{\frac{\rho-1}{\rho}}+(XK)^{\frac{\rho-1}{\rho}}\right)}{\partial\log X}=\frac{\rho-1}{\rho}\left[\frac{(UA)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}}+(XK)^{\frac{\rho-1}{\rho}}}\cdot\frac{\partial\log A}{\partial\log X}+\frac{(XK)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}}+(XK)^{\frac{\rho-1}{\rho}}}\right] \tag{A.18}$$

and that

$$\frac{\partial \log\left((ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}\right)}{\partial \log X}=\frac{\theta-1}{\theta}\left[\frac{(ZM)^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}}\cdot\frac{\partial \log M}{\partial \log X}+\frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}}\cdot\frac{\partial \log V}{\partial \log X}\right] \tag{A.19}$$

Further note that

$$\frac{\partial \log V}{\partial \log X}=\frac{(UA)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}}+(XK)^{\frac{\rho-1}{\rho}}}\cdot\frac{\partial \log A}{\partial \log X}+\frac{(XK)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}}+(XK)^{\frac{\rho-1}{\rho}}} \tag{A.20}$$

Bringing back equation (A.20) to equation (A.19) gives

$$\begin{aligned}\frac{\partial \log\left((ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}\right)}{\partial \log X}=\frac{\theta-1}{\theta}&\left[\frac{(ZM)^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}}\cdot\frac{\partial \log M}{\partial \log X}\right.\\ &+\frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}}\cdot\frac{(UA)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}}+(XK)^{\frac{\rho-1}{\rho}}}\cdot\frac{\partial \log A}{\partial \log X}\\ &\left.+\frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}}\cdot\frac{(XK)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}}+(XK)^{\frac{\rho-1}{\rho}}}\right]\end{aligned} \tag{A.21}$$

Considering equations (A.7) and (A.8), we can simplify notations in equations (A.18) to (A.21) by introducing the following expressions for the gross profit margin GM and the firm operating leverage OL, respectively,

$$\frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}}+V^{\frac{\theta-1}{\theta}}}=\frac{Y-P_M M}{Y}=GM \tag{A.22}$$

$$\frac{(UA)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}}+(XK)^{\frac{\rho-1}{\rho}}}=\frac{P_A A}{Y-P_M M}=OL \tag{A.23}$$

Let $p_1^M=\frac{\partial \log P_M}{\partial \log X}$ and $p_1^A=\frac{\partial \log P_A}{\partial \log X}$ represent the variable input price elasticity and the fixed input price elasticity to the aggregate profitability shock, respectively. The equation system (A.16) and (A.17) can then be rewritten as

$$p_1^M=-\frac{1}{\theta}\cdot GM\cdot\frac{\partial \log M}{\partial \log X}+\frac{1}{\theta}\cdot GM\cdot OL\cdot\frac{\partial \log A}{\partial \log X}+\frac{1}{\theta}\cdot GM(1-OL) \tag{A.24}$$

$$p_1^M-p_1^A=-\frac{1}{\theta}\cdot\frac{\partial \log M}{\partial \log X}+\left(\frac{1}{\rho}-\frac{\theta-\rho}{\rho\theta}\cdot OL\right)\frac{\partial \log A}{\partial \log X}-\frac{\theta-\rho}{\rho\theta}(1-OL) \tag{A.25}$$

Equations (A.24) and (A.25) are two linear equations with two unknowns, $\frac{\partial \log M}{\partial \log X}$ and $\frac{\partial \log A}{\partial \log X}$, so we can solve for a unique set of solutions. In particular, the exposures of fixed inputs A to the aggregate profitability shock X, $\frac{\partial \log A}{\partial \log X}$, can be written as

$$\frac{\partial \log A}{\partial \log X} = \frac{\rho[GM(p_1^M - p_1^A) - p_1^M] + GM(1-OL)}{GM(1-OL)} \tag{A.26}$$

which will be used in deriving the exposure of operating profit to aggregate profitability shock and the condition for operating leverage.

Exposure of operating profit to aggregate profitability shock

Plugging equations (A.7) and (A.8) into equation (A.4), operating profit OP can be written as

$$\begin{aligned}
OP &= \max_{\{M,A\}} \{Y - P_M M - P_A A\} \\
&= \max_{\{M,A\}} \left\{ Y \left(1 - \frac{P_M M}{Y} - \frac{P_A A}{Y} \right) \right\} \\
&= Y \left[1 - \frac{(ZM)^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}}} - \frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}}} \cdot \frac{(UA)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}}} \right] \\
&= Y \left[\frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}}} - \frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}}} \cdot \frac{(UA)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}}} \right] \\
&= Y \cdot \frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}}} \cdot \frac{(XK)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}}}
\end{aligned} \tag{A.27}$$

Plugging the expression of Y from equation (A.3) and the expression of V from equation (A.2) into equation (A.27) gives

$$\begin{aligned}
OP &= \frac{\left[(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1} \cdot \frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}}} \cdot \frac{(XK)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}}} \cdot \left[(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \\
&= \left[(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta-1}} \left[(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right]^{\frac{\theta-\rho}{(\rho-1)\theta}} (XK)^{\frac{\rho-1}{\rho}}
\end{aligned} \tag{A.28}$$

With equation (A.6) of P_A, we can further simplify equation (A.28) as

$$\begin{aligned}
OP &= \underbrace{\left[(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta-1}} \left[(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right]^{\frac{\theta-\rho}{(\rho-1)\theta}} U^{\frac{\rho-1}{\rho}} A^{-\frac{1}{\rho}}}_{=P_A} (XK)^{\frac{\rho-1}{\rho}} U^{\frac{1-\rho}{\rho}} A^{\frac{1}{\rho}} \\
&= P_A (XK)^{\frac{\rho-1}{\rho}} U^{\frac{1-\rho}{\rho}} A^{\frac{1}{\rho}}
\end{aligned} \tag{A.29}$$

Proof of Proposition 1: Operating hedge effect

We can get the following expression for gross profit GP from equations (A.7) and (A.22),

$$GP = Y - P_M M = Y \cdot GM = Y \cdot \frac{V^{\frac{\theta-1}{\theta}}}{(ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}}} \tag{A.30}$$

Rearranging accounting variables to the left-hand-side of equation (A.30) and taking partial derivative of the logarithm of both sides of the equation with respect to $\log X$ yields

$$\frac{\partial \log GP}{\partial \log X} - \frac{\partial \log Y}{\partial \log X} = \frac{\theta-1}{\theta} \cdot \frac{\partial \log V}{\partial \log X} - \frac{\partial \log \left((ZM)^{\frac{\theta-1}{\theta}} + V^{\frac{\theta-1}{\theta}} \right)}{\partial \log X} \tag{A.31}$$

Plugging equations (A.20), (A.21), (A.22), (A.23) to equation (A.31), we have

$$\frac{\partial \log GP}{\partial \log X} - \frac{\partial \log Y}{\partial \log X} = \frac{\theta-1}{\theta}(1-GM)\left[-\frac{\partial \log M}{\partial \log X} + OL \cdot \frac{\partial \log A}{\partial \log X} + (1-OL) \right] \tag{A.32}$$

By comparing equation (A.32) and equation (A.24), we can simplify the condition for operating hedge as below

$$\frac{\partial \log GP}{\partial \log X} - \frac{\partial \log Y}{\partial \log X} = (\theta-1)\frac{1-GM}{GM} p_1^M \tag{A.33}$$

We can rearrange equation (A.5) so that

$$P_M = \left[1 + \left(\frac{V}{ZM} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta-1}} Z$$

Then we have

$$\left(\frac{V}{ZM} \right)^{\frac{\theta-1}{\theta}} = \left(\frac{Z}{P_M} \right)^{1-\theta} - 1 \tag{A.34}$$

Plugging equation (A.34) into equation (A.7) gives us an expression of GM

$$GM = 1 - \frac{P_M M}{Y} = \frac{\left(\frac{V}{ZM}\right)^{\frac{\theta-1}{\theta}}}{1 + \left(\frac{V}{ZM}\right)^{\frac{\theta-1}{\theta}}} = 1 - \left(\frac{Z}{P_M} \right)^{\theta-1} \tag{A.35}$$

Proof of Proposition 2: Operating leverage effect

Plugging equation (A.30) into equation (A.27) gives the following expression for operating profit OP,

$$OP = GP \cdot \frac{(XK)^{\frac{\rho-1}{\rho}}}{(UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}}} \tag{A.36}$$

Rearranging accounting variables to the left-hand-side of equation (A.36) and taking partial derivative of the logarithm of both sides of the equation with respect to $\log X$ yields

$$\frac{\partial \log OP}{\partial \log X} - \frac{\partial \log GP}{\partial \log X} = \frac{\rho - 1}{\rho} - \frac{\partial \log \left((UA)^{\frac{\rho-1}{\rho}} + (XK)^{\frac{\rho-1}{\rho}} \right)}{\partial \log X} \tag{A.37}$$

Plugging equations (A.15), (A.18), (A.22), (A.23)and (A.26) into equation (A.37), we have

$$\frac{\partial \log OP}{\partial \log X} - \frac{\partial \log GP}{\partial \log X} = (1-\rho)\frac{OL}{1-OL}\left(p_1^M - p_1^A - \frac{p_1^M}{GM}\right) \tag{A.38}$$

Proof of Proposition 3: Firm risk exposure to the aggregate profitability shock

Taking partial derivative of the logarithm of both sides of equation (A.29) with respect to $\log X$ yields

$$\frac{\partial \log OP}{\partial \log X} = \frac{\partial \log P_A}{\partial \log X} + \frac{1}{\rho} \cdot \frac{\partial \log A}{\partial \log X} + \frac{\rho - 1}{\rho} \cdot \frac{\partial \log K}{\partial \log X} \tag{A.39}$$

Plugging equations (A.15) and (A.26) into equation (A.39), we arrive at a firm's risk exposure to the aggregate profitability shock as follows

$$\beta = \frac{\partial \log OP}{\partial \log X} = 1 + p_1^A + \frac{1}{1-OL}\left(p_1^M - p_1^A - \frac{p_1^M}{GM}\right) \tag{A.40}$$

Stock return predictors used in the paper

The table below lists the 140 stock return predictors from Chen and Zimmermann (2021) used in this paper to construct the neural network measure of operating leverage. We follow their description in our paper.

Predictor	Description	Predictor	Description
Accruals	Accruals	HerfBE	Industry concentration (equity)
AccrualsBM	Book-to-market and accruals	High52	52 week high
AdExp	Advertising expense	IdioVol3F	Idiosyncratic risk (3 factor)
AM	Total assets to market	IdioVolAHT	Idiosyncratic risk (AHT)
AssetGrowth	Asset growth	Illiquidity	Amihud's illiquidity
Beta	CAPM beta	IndIPO	Initial public offerings
BetaFP	Frazzini–Pedersen beta	IndMom	Industry momentum
BetaTailRisk	Tail risk beta	IndRetBig	Industry return of big firms
BidAskSpread	Bid-ask spread	IntanCFP	Intangible return using CFtoP
BM	Book to market using most recent ME	IntanEP	Intangible return using EP
BMdec	Book to market using December ME	IntanSP	Intangible return using Sale2P
BookLeverage	Book leverage (annual)	IntMom	Intermediate momentum
BPEBM	Leverage component of BM	Investment	Investment to revenue
CashProd	Cash productivity	InvestPPEInv	Change in ppe and inv/assets
CBOperProf	Cash-based operating profitability	InvGrowth	Inventory growth
CF	Cash flow to market	Leverage	Market leverage
cfp	Operating cash flows to price	LRreversal	Long-run reversal
ChAssetTurnover	Change in asset turnover	MaxRet	Maximum return over month
ChEQ	Growth in book equity	MeanRankRevGrowth	Revenue growth rank
ChInv	Inventory growth	Mom12m	Momentum (12 month)
ChInvIA	Change in capital inv (ind adj)	Mom12mOffSeason	Momentum without the seasonal part
ChNNCOA	Change in net noncurrent op assets	Mom6m	Momentum (6 month)
ChNWC	Change in net working capital	MomOffSeason	Off season long-term reversal
ChTax	Change in taxes	MomOffSeason06YrPlus	Off season reversal years 6 to 10
CompEquIss	Composite equity issuance	MomOffSeason11YrPlus	Off season reversal years 11 to 15
CompositeDebtIssuance	Composite debt issuance	MomOffSeason16YrPlus	Off season reversal years 16 to 20
ConvDebt	Convertible debt indicator	MomRev	Momentum and LT reversal
CoskewACX	Coskewness using daily returns	MomSeason	Return seasonality years 2 to 5
Coskewness	Coskewness	MomSeason06YrPlus	Return seasonality years 6 to 10
DebtIssuance	Debt issuance	MomSeason11YrPlus	Return seasonality years 11 to 15
DelCOA	Change in current operating assets	MomSeason16YrPlus	Return seasonality years 16 to 20
DelCOL	Change in current operating liabilities	MomSeasonShort	Return seasonality last year
DelEqu	Change in equity to assets	MomVol	Momentum in high volume stocks
DelFINL	Change in financial liabilities	MRreversal	Medium-run reversal
DelLTI	Change in long-term investment	MS	Mohanram G-score
DelNetFin	Change in net financial assets	NetDebtPrice	Net debt to price
DivInit	Dividend initiation	NetPayoutYield	Net payout yield
DivOmit	Dividend omission	NOA	Net operating assets
DivSeason	Dividend seasonality	NumEarnIncrease	Earnings streak length
DivYieldST	Predicted div yield next month	OperProf	operating profits/book equity
dNoa	change in net operating assets	OperProfRD	Operating profitability R&D adjusted
DolVol	Past trading volume	OPLeverage	Operating leverage
EarningsConsistency	Earnings consistency	OrgCap	Organizational capital
EarningsSurprise	Earnings surprise	PayoutYield	Payout yield
EarnSupBig	Earnings surprise of big firms	PctAcc	Percent operating accruals
EBM	Enterprise component of BM	PriceDelayRsq	Price delay r square
EntMult	Enterprise multiple	PriceDelaySlope	Price delay coeff
EP	Earnings-to-price ratio	PriceDelayTstat	PriceDelayTstat
EquityDuration	Equity duration	RD	R&D over market cap
ExchSwitch	Exchange switch	RDAbility	R&D ability
FirmAge	Firm age based on CRSP	RDIPO	IPO and no R&D spending
Frontier	Efficient frontier index	RealizedVol	Realized (Total) Volatility
GP	gross profits/total assets	ResidualMomentum	Momentum based on FF3 residuals
grcapx	Change in capex (two years)	ReturnSkew	Return skewness
grcapx3y	Change in capex (three years)	ReturnSkew3F	Idiosyncratic skewness (3F model)
GrLTNOA	Growth in long term operating assets	RevenueSurprise	Revenue surprise
GrSaleToGrInv	Sales growth over inventory growth	RIO_MB	Inst own and market to book
GrSaleToGrOverhead	Sales growth over overhead growth	RIO_Turnover	Inst own and turnover
Herf	Industry concentration (sales)	RIO_Volatility	Inst own and idio vol
HerfAsset	Industry concentration (assets)	RoE	Net income/book equity

Stock return predictors used in the paper (continued)

Predictor	Description
ShareIss1Y	Share issuance (1 year)
ShareIss5Y	Share issuance (5 year)
ShareRepurchase	Share repurchases
ShareVol	Share volume
sinAlgo	Sin stock (selection criteria)
SP	Sales-to-price
Spinoff	Spinoffs
std_turn	Share turnover volatility
SurpriseRD	Unexpected R&D increase
tang	Tangibility
Tax	Taxable income to income
TotalAccruals	Total accruals
TrendFactor	Trend factor
VarCF	Cash-flow to price variance
VolMkt	Volume to market equity
VolSD	Volume variance
VolumeTrend	Volume trend
zerotrade	Days with zero trades
zerotradeAlt1	Days with zero trades
zerotradeAlt12	Days with zero trades

Table 1: Cyclicality of gross profits, operating profits, COGS, and SG&A

This table reports the results of time series regressions in which annual growth rate of aggregate gross profits (Δ logGP), aggregate operating profits (Δ logOP), aggregate cost of good sold (Δ logCOGS), and aggregate selling, general, and administrative expenses (Δ logXSGA) are regressed on the annual growth aggregate revenue (Δ logREVT). All growth rates are adjusted for inflation. The sample period is from 1963 to 2019.

	Δ logGP	Δ logOP	Δ logCOGS	Δ logXSGA
Intercept	1.28	-0.54	-0.57	3.53
	(3.14)	(-0.58)	(-3.11)	(8.14)
Δ logREVT	0.88	1.41	1.05	0.48
	(14.12)	(9.97)	(37.45)	(7.21)
R^2	78.7%	64.8%	96.3%	49.1%

Table 2: Parameter values in model calibration

This table reports the parameter values used in model calibration at the annual frequency.

Symbol	Parameter description	Value
ρ	Elasticity of substitution b/w physical capital (K) and fixed inputs (A)	0.47
θ	Elasticity of substitution b/w K-A bundle and variable inputs (M)	0.74
x_{min}	The minimum value of aggregate profitability shock	1.91
x_{max}	The maximum value of aggregate profitability shock	1.93
μ_z	Mean of firm-level variable input productivity	2.45
σ_z	Standard deviation of firm-level variable input productivity	0.91
μ_u	Mean of firm-level fixed input productivity	1.45
σ_u	Standard deviation of firm-level fixed input productivity	0.42
P_M^0	Level of price of variable inputs	0.44
P_M^1	Elasticity of variable input price w.r.t. aggregate profitability shock	1.39
P_A^0	Level of price of fixed inputs	0.26
P_A^1	Elasticity of fixed input price w.r.t. aggregate profitability shock	0.45
λ	Risk premium of aggregate profitability shocks	0.09

Table 3: Correlations of operating leverage measures and firm characteristics

This table reports the correlation matrix of eight measures of operating leverage. The eight measures include our flow-based operating leverage (OL_{FL}), the neural network measure (OL_{NN}), the operating leverage defined in Novy-Marx (2011) (OL_{NM}, the sum of COGS and SG&A divided by AT), Ferri and Jones (1979) (OL_{FJ}, PPENT divided by AT), Chen, Hartford, and Kamara (2019) (OL_{CHK}, SG&A divided by AT), and Chen, Chen, Li, and Li (2021) (OL_{CCLL}, the sum of DP and SG&A divided by market value of assets). The characteristics reported include logarithm of book-to-market ratio (logBM), gross profitability (GPA), logarithm of June-end market capitalization (logME), and annualized idiosyncratic volatility (IVOL). The sample period is from 1963 to 2020.

Panel A: Correlations of operating leverage measures

	OL_{FL}	OL_{NN}	OL_{NM}	OL_{FJ}	OL_{CHK}	OL_{CCLL}
OL_{FL}	1	0.6782	0.3234	-0.4040	0.6123	0.5300
OL_{NN}	0.6782	1	0.0923	-0.1152	0.2875	0.3239
OL_{NM}	0.3234	0.0923	1	-0.1988	0.5378	0.4731
OL_{FJ}	-0.4040	-0.1152	-0.1988	1	-0.3200	-0.1487
OL_{CHK}	0.6123	0.2875	0.5378	-0.3200	1	0.7081
OL_{CCLL}	0.5300	0.3239	0.4731	-0.1487	0.7081	1

Panel B: Correlations with firm characteristics

	OL_{FL}	OL_{NN}	logME	logBM	GPA	IVOL
OL_{FL}	1	0.6782	-0.3538	0.0776	0.3373	0.2339
OL_{NN}	0.6782	1	-0.3220	0.1239	0.0356	0.2517
logME	-0.3538	-0.3220	1	-0.3522	-0.0562	-0.4685
logBM	0.0776	0.1239	-0.3522	1	-0.2707	0.0880
GPA	0.3373	0.0356	-0.0562	-0.2707	1	0.0018
IVOL	0.2339	0.2517	-0.4685	0.0880	0.0018	1

Table 4: Elasticities of operating profits

This table reports the results of the panel regressions of percentage change in firm-level operating profit on percentage change in firm-level gross profit and its interaction with our flow-based operating leverage (OL_{FL}), neural network measure (OL_{NN}), the operating leverage defined in Novy-Marx (2011) (OL_{NM}, the sum of COGS and SG&A divided by AT), Ferri and Jones (1979) (OL_{FJ}, PPENT divided by AT), Chen, Hartford, and Kamara (2019) (OL_{CHK}, SG&A divided by AT), and Chen, Chen, Li, and Li (2021) (OL_{CCLL}, the sum of DP and SG&A divided by market value of assets). We normalize OL, OL_{NN}, OL_{NM}, OL_{FJ}, OL_{CHK}, OL_{CCLL} to have the unit standard deviation. Firm-, industry-, and year-fixed effects are applied in all specifications. We report t-statistics with firm clustering and year clustering. Variables are winsorized at 1% and 99%. The sample period is from fiscal year 1963 to 2020.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
%GP	4.96	3.16	2.10	4.97	4.92	4.76	4.81	3.17	3.16	3.18	3.19	3.17	2.09	2.10	2.09	2.09	2.09	2.10	2.09
	(17.4)	(18.1)	(50.9)	(17.4)	(17.4)	(17.3)	(18.2)	(17.7)	(18.2)	(18.2)	(19.0)	(19.0)	(50.4)	(51.0)	(51.1)	(49.7)	(51.3)	(65.0)	(63.6)
$\%GP \times OL_{FL}$		1.14						1.14	1.14	1.09	1.08	1.07						-0.01	0.00
		(11.0)						(10.8)	(10.7)	(10.3)	(10.6)	10.2						(-0.2)	(0.0)
$\%GP \times OL_{NN}$			1.75										1.76	1.75	1.77	1.76	1.76	1.76	1.76
			(41.9)										(42.1)	(41.6)	(40.2)	(40.6)	(39.6)	(32.5)	(31.7)
$\%GP \times OL_{NM}$				0.33				0.08				-0.17	-0.03				0.01		0.01
				(6.9)				(1.6)				(-3.7)	(-1.6)				(0.5)		(0.5)
$\%GP \times OL_{FJ}$					-0.35				-0.08			-0.03		0.01			-0.01		-0.01
					(-6.0)				(-1.3)			(-0.6)		(0.6)			(-0.9)		(-0.9)
$\%GP \times OL_{CHK}$						0.67				0.22		0.03			-0.05		-0.06		-0.06
						(10.3)				(3.5)		(0.3)			(-2.7)		(-2.3)		(-2.4)
$\%GP \times OL_{CCLL}$							0.75				0.38	0.43				-0.03	0.00		0.00
							(10.3)				(5.7)	(5.4)				(-1.7)	(0.2)		(0.2)
$R^2(\%)$	70.5	78.8	90.3	70.8	70.8	72.1	72.3	78.8	78.8	78.9	79.2	79.3	90.3	90.3	90.3	90.3	90.3	90.3	90.3

Table 5: Excess returns of portfolios double sorted by GPA and OL

This table reports average annualized value-weighted excess returns of portfolios double sorted on GPA and flow-based operating leverage (OL_{FL}) in Panel A, and portfolios double sorted on gross profitability (GPA) and neural network measure of operating leverage (OL_{NN}) in Panel B. Newey-West t-statistics reported in parentheses control for autocorrelation and heteroskedasticity. The sample period is from July 1964 to June 2020.

Panel A: Double sorted portfolios of GPA and OL_{FL}

1) Sequentially sort by OL_{FL} first and then GPA

	1	2	GPA	4	5	5-1	t-stat
1	3.80	4.59	5.95	6.93	6.42	2.62	(1.32)
OL_{FL}	6.70	7.06	7.13	7.36	9.28	2.58	(1.41)
3	1.73	6.25	8.82	8.71	10.50	8.78	(2.96)

2) Sequentially sort by GPA first and then OL_{FL}

	1	2	OL_{FL}	4	5	5-1	t-stat
1	3.90	4.15	5.25	4.87	0.76	-3.13	(-0.83)
GPA	7.07	6.97	7.27	8.30	7.41	0.35	(0.12)
3	7.12	7.89	10.41	11.50	11.12	4.00	(1.19)

Panel B: Double sorted portfolios of GPA and OL_{NN}

1) Sequentially sort by OL_{NN} first and then GPA

	1	2	GPA	4	5	5-1	t-stat
1	3.31	7.01	5.49	6.37	9.36	6.04	(3.27)
OL_{NN}	4.15	7.75	6.92	7.58	10.69	6.54	(3.04)
3	1.86	3.86	7.28	10.74	11.31	9.45	(3.42)

2) Sequentially sort by GPA first and then OL_{NN}

	1	2	OL_NN	4	5	5-1	t-stat
1	3.87	3.91	5.59	3.84	2.20	-1.67	(-0.57)
GPA	5.73	6.41	8.79	9.36	9.04	3.32	(1.59)
3	7.64	7.86	6.98	11.52	11.29	3.64	(1.78)

Table 6: Portfolio excess returns

This table reports the asset pricing results of 10 portfolios sorted on gross profitability (GPA) in Panel A, flow-based operating leverage (OL_{FL}) in Panel B, and neural network measure of operating leverage (OL_{NN}) in Panel C. Newey-West t-statistics reported in parentheses control for autocorrelation and heteroskedasticity. The sample period is from July 1964 to June 2020.

Panel A: 10 GPA portfolios

GPA decile	Ret - RF	α_{CAPM}	β_{CAPM}	α_{FF3F}	β_{MKT}	β_{SMB}	β_{HML}
1	2.53	-5.43	1.26	-6.49	1.23	0.30	0.19
2	3.23	-3.44	1.06	-4.02	1.09	-0.04	0.17
3	5.73	-0.67	1.02	-1.20	1.04	-0.03	0.15
4	7.74	1.33	1.02	1.10	1.00	0.11	0.03
5	6.59	0.18	1.02	0.40	0.99	0.07	-0.08
6	6.85	0.09	1.07	0.97	1.04	0.01	-0.24
7	7.27	0.82	1.02	1.60	0.98	0.05	-0.22
8	6.03	-0.39	1.02	0.70	0.99	-0.05	-0.27
9	9.47	3.73	0.91	4.85	0.89	-0.07	-0.28
10	8.81	2.52	1.00	3.40	0.95	0.07	-0.26
Hi-Lo	6.28	7.95	-0.27	9.90	-0.28	-0.22	-0.45
t-stat	(2.77)	(3.50)	(-4.97)	(4.75)	(-5.97)	(-3.80)	(-4.96)

Panel B: 10 OL_{FL} portfolios

OL_{FL} decile	Ret - RF	α_{CAPM}	β_{CAPM}	α_{FF3F}	β_{MKT}	β_{SMB}	β_{HML}
1	5.51	-0.56	0.96	-0.68	0.99	-0.08	0.06
2	5.77	-0.56	1.01	-0.61	1.03	-0.09	0.04
3	6.66	0.35	1.00	0.95	0.98	0.00	-0.16
4	6.87	0.79	0.97	1.41	0.95	-0.05	-0.15
5	7.99	1.44	1.04	2.07	1.00	0.07	-0.19
6	8.10	1.52	1.04	2.03	1.01	0.05	-0.15
7	8.95	1.69	1.15	2.25	1.07	0.26	-0.23
8	9.95	2.00	1.26	2.83	1.12	0.47	-0.36
9	6.92	-1.81	1.39	-0.76	1.17	0.70	-0.49
10	0.95	-8.46	1.50	-8.31	1.25	0.96	-0.33
Hi-Lo	-4.56	-7.90	0.53	-7.63	0.26	1.05	-0.39
t-stat	(-1.37)	(-2.61)	(7.41)	(-2.96)	(4.10)	(13.50)	(-3.29)

Panel C: 10 OL_{NN} portfolios

OL_{NN} decile	Ret - RF	α_{CAPM}	β_{CAPM}	α_{FF3F}	β_{MKT}	β_{SMB}	β_{HML}
1	5.98	-0.37	1.01	0.40	1.00	-0.08	-0.18
2	5.79	-0.04	0.93	0.09	0.94	-0.09	-0.01
3	6.63	0.46	0.98	0.81	0.98	-0.06	-0.08
4	5.87	-0.57	1.02	-0.30	1.01	0.00	-0.07
5	6.09	-0.49	1.04	-0.26	1.03	0.03	-0.07
6	8.89	2.11	1.08	2.19	1.05	0.10	-0.05
7	9.69	2.49	1.14	2.57	1.06	0.32	-0.12
8	8.46	1.31	1.14	1.46	1.07	0.26	-0.12
9	8.28	0.27	1.27	0.10	1.16	0.46	-0.09
10	4.15	-4.68	1.40	-4.10	1.23	0.62	-0.34
Hi-Lo	-1.83	-4.31	0.39	-4.50	0.23	0.70	-0.16
t-stat	(-0.73)	(-1.85)	(6.60)	(-2.08)	(4.19)	(7.69)	(-1.71)

Table 7: Excess returns of portfolios sequentially sorted by OL and BM

This table reports characteristics of BM quintile portfolios in Panel A, and average annualized value-weighted excess returns of portfolios sequentially sorted first by operating leverage and then by book-to-market ratio (BM) in Panels B and C. Our flow-based operating leverage (OL_{FL}) is used in Panel B, and the neural network measure of operating leverage (OL_{NN}) is used in Panel C. Newey-West t-statistics reported in parentheses control for autocorrelation and heteroskedasticity. The sample period is from July 1964 to June 2020.

Panel A: BM quintile portfolios

BM quintile	Ret-Rf	OL_{NN}	OL_{FL}	OL_{NM}	OL_{FJ}	OL_{CHK}	OL_{CCLL}	logBM	GPA	logME	IVOL
1	5.70	0.66	1.29	0.99	0.21	0.33	0.11	-1.62	0.47	5.41	0.36
2	6.23	0.63	1.25	1.02	0.23	0.26	0.17	-0.88	0.41	5.32	0.32
3	7.17	0.64	1.30	1.06	0.24	0.23	0.21	-0.45	0.37	4.95	0.32
4	8.17	0.67	1.39	1.09	0.25	0.22	0.26	-0.07	0.34	4.40	0.34
5	9.43	0.73	1.53	1.12	0.27	0.21	0.33	0.47	0.30	3.52	0.41
5-1	3.73	0.07	0.24	0.13	0.06	-0.11	0.22	2.09	-0.18	-1.89	0.04
t-stat	(1.69)										

Panel B: Double sorted portfolios of OL_{FL} and BM

	1	2	BM	4	5	5-1	t-stat
1	4.26	5.45	5.90	6.72	7.40	3.14	(1.39)
OL_{FL}	7.40	7.83	9.42	9.16	12.00	4.60	(1.94)
3	2.97	6.66	9.39	9.90	10.05	7.08	(2.04)
			Conditional value premium			4.94	(3.73)

Panel C: Double sorted portfolios of OL_{NN} and BM

	1	2	BM	4	5	5-1	t-stat
1	5.80	6.02	5.86	6.40	7.54	1.75	(0.78)
OL_{NN}	6.53	5.62	9.01	9.44	9.00	2.47	(1.04)
3	2.38	7.50	10.36	9.02	9.22	6.83	(2.27)
			Conditional value premium			3.69	(2.84)

Figure 1: Value and policy functions

This figure plots the optimal policies for fixed input (A) and variable input (M), gross profitability (GP/A), operating leverage (OL), gross margin (GM), and operating profitability (OP/A), against fixed input productivity (u) and variable input productivity (z).

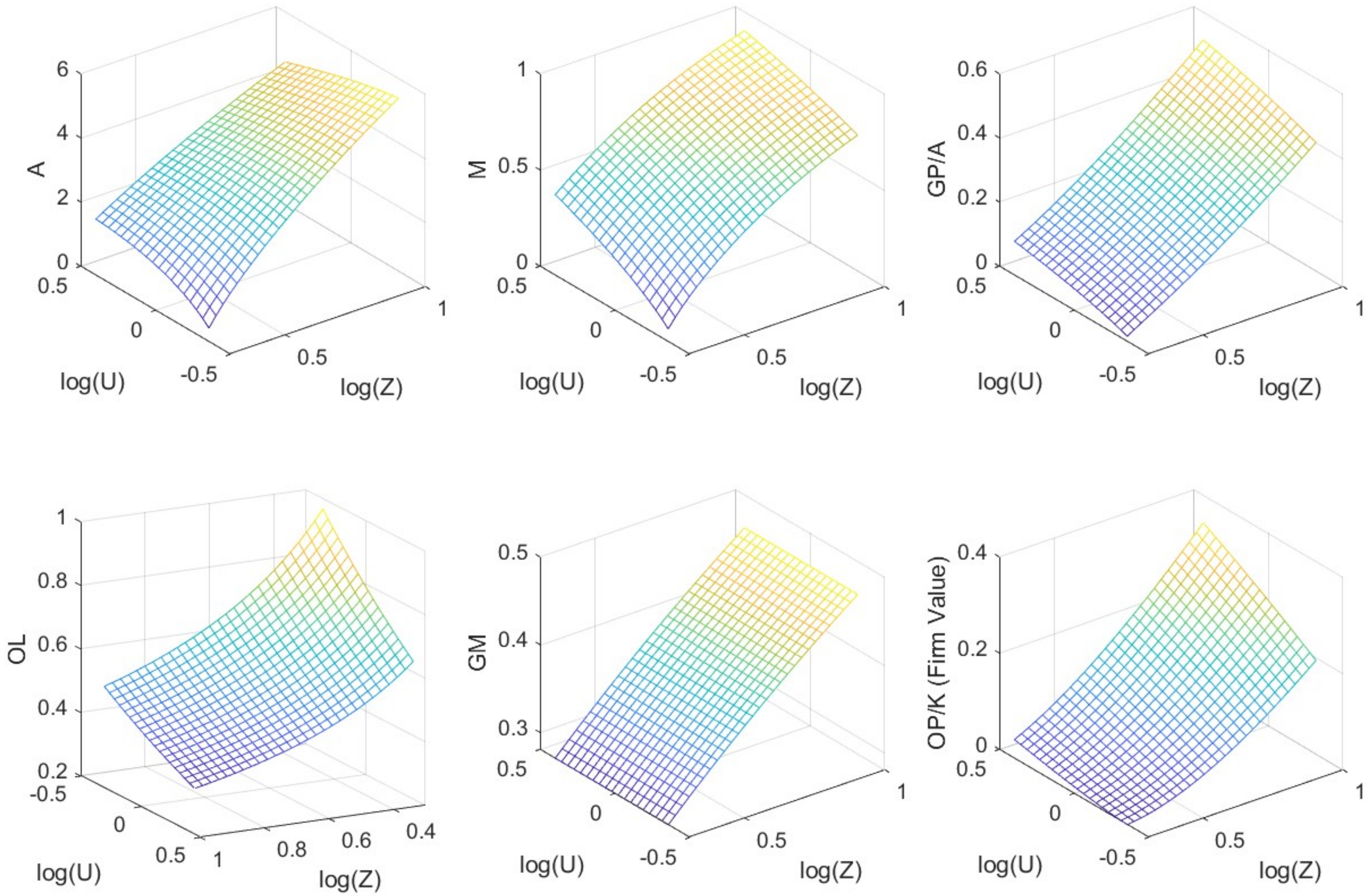

Figure 2: Risk exposures

This figure plots firm's exposure to the aggregate profitability shock (beta) against the fixed input productivity (u) and the variable input productivity (z) in Panel A, and against gross profitability (GP/A) and operating leverage (OL) in Panel B.